R. Beck and H.-P. Krahn: Constraining the (γ, π) amplitude for $E2$ $N \to \Delta$

In a recent Letter [1] we have reported precision measurements of differential cross sections and polarized photon asymmetries for the reaction $\vec{\gamma}p \to p\pi^0$ with the DAPHNE–detector, using tagged photons at the Mainz Microtron MAMI. The above Comment [2] critizies our value $R_{EM} = ImE_{1+}^{3/2}/ImM_{1+}^{3/2} = -(2.5 \pm 0.2 \pm 0.2)\%$, because of possible ambiguities stemming from contributions of higher partial waves.

We are using Eqs. (3) to (7) in our paper [1] to extract the R_{EM} value. These Eqs. are exact under the assumption that only s– and p–waves contribute. To study the validity of this assumption we investigated the effects of higher partial waves ($l_\pi \geq 2$) in Eqs. 3 to 7. The inclusion of d–waves results in a modification of Eq. (3) to

$$\frac{d\sigma}{d\Omega} = \frac{q}{k}(A + B\cos(\theta) + C\cos^2(\theta) + D\cos^3(\theta) + E\cos^4(\theta)) \ . \tag{1}$$

Two additional coefficients D and E appear and furthermore the coefficients A, B and C are modified according to

$$A \simeq A(s_{wave}, p_{wave}) + \mathrm{Re}\left[E_{0+}d_{wave}^*\right] + |d_{wave}|^2 \ , \tag{2}$$

$$B \simeq B(s_{wave}, p_{wave}) + \mathrm{Re}\left[(M_{1+} - M_{1-})d_{wave}^*\right] \ , \tag{3}$$

$$C \simeq C(s_{wave}, p_{wave}) + \mathrm{Re}\left[E_{0+}d_{wave}^*\right] + |d_{wave}|^2 \ , \tag{4}$$

$$D \simeq \mathrm{Re}\left[(M_{1+} - M_{1-})d_{wave}^*\right] \ , \tag{5}$$

$$E = |d_{wave}|^2 \ , \tag{6}$$

where s_{wave}, p_{wave} and d_{wave} are combinations of the corresponding partial wave multipoles. The effect is largest for the coefficients B and D, where an interference term between the large M_{1+} and the d–waves occurs. But at the top of the resonance ($\delta_{33} = 90^0$) the contributions of these terms can be neglected, e.g.

$$\mathrm{Re}\left[(M_{1+} - M_{1-})E_{2-}^*\right] = \mathrm{Re}(M_{1+} - M_{1-})\mathrm{Re}E_{2-} + \mathrm{Im}(M_{1+} - M_{1-})\mathrm{Im}E_{2-} \quad . \tag{7}$$

The first term vanishes, because $\mathrm{Re}(M_{1+} - M_{1-})$ goes through zero near the resonance energy ($E_\gamma = 340$ MeV) and the second term can be neglected, because $\mathrm{Im}E_{2-}$ is small

due to a phase close to zero. Fig. 1 shows the ratio of the differential cross section for only $s-$ and $p-$waves contribution to the cross section where higher partial waves have been taken into account (truncation at $f-$waves, Born contribution for $l_\pi \geq 4$) VPI[SM95] [3]. The ratio is shown at $\theta_\pi = 0^0$, 90^0 and 180^0 in the energy region of 200 to $500 MeV$. At $\theta_\pi = 90^0$, the contributions from the higher partial waves are far below 1%, since there is only an interference term with the $s-$wave E_{0+} (e.g. $Re(E_{0+}d^*_{wave})$). Below and above the resonance, however, contributions from $l_\pi \geq 2$ are of the order of $10-20\%$ of the differential cross section at 0^0 and 180^0. This will affect the $C_{\parallel}-$coefficient below and above the resonance.

Another observable which is sensitive to a contribution of higher partial waves is the linear polarization cross section difference $d\sigma_\perp - d\sigma_{\parallel}$. Fig. 2 shows $d\sigma_\perp - d\sigma_{\parallel}$ in a power series expansion in $cos\theta$ for our (p, π^0) data

$$d\sigma_\perp - d\sigma_{\parallel} = \Sigma d\sigma / sin^2\theta = \frac{q}{k}(A_\Sigma + B_\Sigma cos\theta + C_\Sigma cos^2\theta) \tag{8}$$

with

$$A_\Sigma \simeq A(s_{wave}, p_{wave}) + \text{Re}\left[E_{0+}d^*_{wave}\right] + |d_{wave}|^2 , \tag{9}$$

$$B_\Sigma \simeq \text{Re}\left[(M_{1+} - M_{1-})d^*_{wave}\right] , \tag{10}$$

$$C_\Sigma = |d_{wave}|^2 . \tag{11}$$

In the case, where only $s-$ and $p-$waves contribute, this difference should be equal to A_Σ and therefore constant, independent of the pion angle θ_π. The $B_\Sigma-$coefficient is an interference term between the large $M_{1+}-$amplitude and the $d-$waves. There are NO ambiguities and NO indications for a non–Born contribution for higher partial waves $l_\pi \geq 2$ around the $\Delta(1232)-$resonance in our (p, π^0) data.

In Table 1 of reference [2], we believe that the LEGS analysis is running into the classical problem of a multipole analysis: How to handle systematic errors coming from different experiments? It is certainly not reasonable to increase the number of partial waves until the fit is stable, because there is already systematics absorbed into the partial waves. One

has instead to look at observables, which are sensitive to the d–wave contribution. This has already been pointed out by the original multipole analysis of the Khark'hov data [4]. In this work it was demonstrated, that there is no need of higher partial waves (non–Born contribution) in the polarization observables (Σ, T and P). There is, as correctly pointed out in [4], a definite incompatibility of the experimental Bonn data on $d\sigma$ at the extreme forward and backward angles to the photon asymmetry result $\Sigma(90^0)$ from Khark'hov. This is very important, because these two observables have a similar $M_{1+}E_{1+}$ interference term in this angular range.

In conclusion, there are *NO* ambiguities stemming from necglecting contributions of higher partial waves in our (p, π^0) analysis and there is *NO* reason to change our value $R_{EM} = Im E_{1+}^{3/2}/Im M_{1+}^{3/2} = -(2.5 \pm 0.2 \pm 0.2)\%$.

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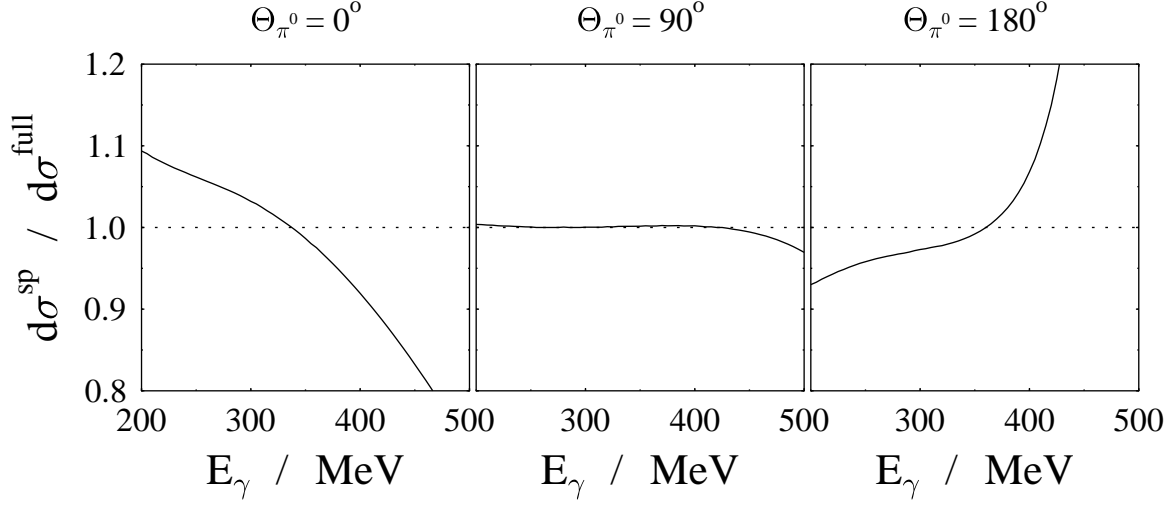
FIG. 1.   The ratio of the differential cross section for only $s-$ and $p-$waves contributions to the cross section where higher partial waves have been taken into account (truncation at $f-$waves, Born contribution for $l_\pi \geq 4$) VPI[SM95].
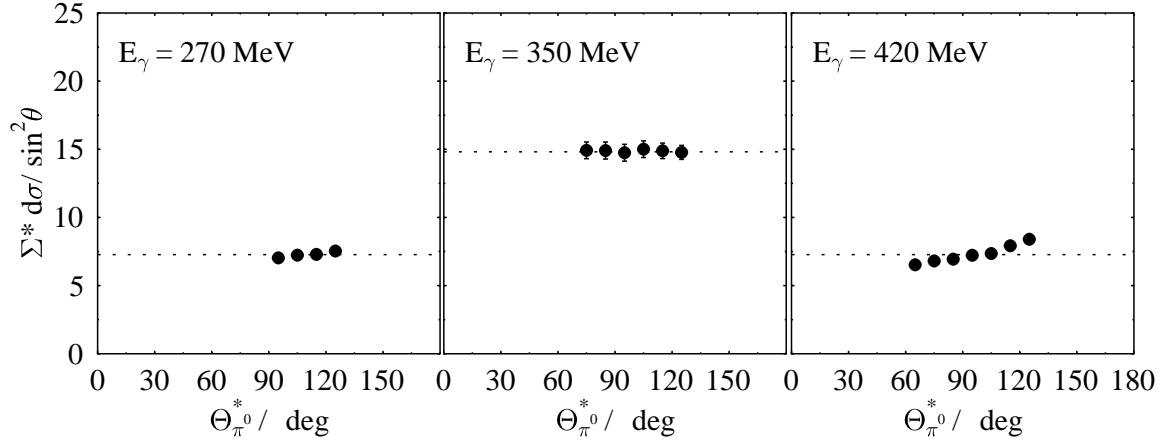

FIG. 2. The linear polarization cross section difference $d\sigma_\perp - d\sigma_{||} = \Sigma d\sigma/sin^2\theta$ for $p(\vec{\gamma}, p)\pi^0$.